

Modeling with Mathematics - Fractions Bridging the Transition from Elementary to Middle School

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Warm-Up: Ballpark Fractions

$\frac{1}{8} \qquad \frac{1}{4} \qquad \frac{1}{2} \qquad \frac{3}{4} \qquad 1$

Estimate which fraction benchmark best describes the people in the room.

Had at least 1 cup of coffee this morning Traveled more than 1000 miles away this summer Are left handed

Have a dog

Building Fraction Sense with Fraction Strips Equivalence, Comparing, Ordering, Adding, Subtracting

Together:			
Make the Kit	Questions to think about	What math are students getting from each experience?	
Cover Up			
Uncover Version 1 & 2		What questions might you ask?	
Menu		How do these experiences lay a foundation for the fraction	
Pick 2		understanding students need at	
Pick 2 Pick 3		understanding students need at your grade level?	
Pick 2 Pick 3 Roll 5		understanding students need at your grade level?	

Make a Whole

Leveraging Number Talks & Modeling to Generalize





The Number Line – A Critical Visual Model

A number line representation of number quantity has been shown in cognitive studies to be particularly important for the development of numerical knowledge and a precursor of children's academic success (Kucian et al., 2011; Hubbard et al., 2005, Booth & Siegler 2004; Schneider et al., 2009).

--Boaler, J. 2016



How might the number line help students make sense of this problem?

$$\frac{1}{2} + \frac{3}{8} =$$

How will you partition the number line? Why? How might this model support student thinking about common denominators?

Common Core Standards Fraction Progression

	8			
Grade 4	Grade 5	Grade 6	Grade 7	
Extend understanding of fraction	Use equivalent fractions as a strategy	Apply and extend previous	Apply and extend previous	
equivalence and ordering using	for adding and subtracting fractions	understandings of multiplication and	understandings of operations with	
visual fraction models.	(with unlike denominators, including mixed numbers) using visual fraction	division to divide fractions by fractions.	fractions to add, subtract, multiply, and divide rational numbers.	
Build fractions from unit fractions	models and equations.			
by applying and extending		Interpret and compute quotients of		
previous understandings of	Apply and extend previous	fractions, and solve word problems		
operations on whole numbers	understandings of multiplication and	involving division of fractions by		
using visual fraction models and	division to multiply and divide	fractions using visual fraction		
equations.	fractions using visual fraction models	models, equations, and the		
	or equations. (multiply a fraction or	relationship between multiplication		
Add and subtract fractions and	whole number by a fraction; divide	and division to explain and represent		
mixed numbers with like	unit fractions by whole numbers and	problems.		
denominators, using visual	whole numbers by unit fractions)			
fraction models and equations.				
Apply previous understandings of				
multiplication to multiply a				
fraction by a whole number, using				
visual traction models and				
equations.				

Examining Common Errors





How can we help students make sense of fraction operations?





Modeling with Fraction Strips

MATH TRANSFORMATIONS

Do "groups of" and repeated addition apply when multiplying a whole number by a fraction? Does the commutative property apply?

 $3 \times \frac{1}{4} = \frac{1}{4} \times 3 = 2\frac{1}{2} \times 2 =$ $3 \times \frac{1}{2} = \frac{1}{2} \times 3 = 2\frac{1}{2} \times \frac{1}{2} =$ $3 \times \frac{3}{16} = \frac{3}{16} \times 3 = \frac{3}{4} \times 2\frac{1}{2} =$ $6 \times \frac{3}{8} = \frac{3}{8} \times 6 = 2\frac{1}{2} \times \frac{3}{8} =$

How might you model any of the problems on this slide with the number line?

Language Support: _____ groups of _____.

Challenges

Do "groups of" and/or repeated addition apply when multiplying a fraction by a fraction?

$$\frac{1}{2} \times \frac{1}{4}$$

 $\frac{1}{2}$ group of $\frac{1}{4}$



$$\frac{1}{2}$$
 of $\frac{1}{4}$

How can you model these problems?

What do you notice about this string of problems?



How can the first problem help you solve the second problem in the pairs?

Challenges



CC Fraction Multiplication Standards

4th Grade

CCSS.MATH.CONTENT.4.NF.B.4.C

Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

5th Grade

CCSS.MATH.CONTENT.5.NF.B.4

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

CCSS.MATH.CONTENT.5.NF.B.5

Interpret multiplication as scaling (resizing), ...

CCSS.MATH.CONTENT.5.NF.B.6

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Division with Fractions



Division with Fractions

What do you know about division with whole numbers? How might that knowledge help you with dividing fractions?

 $20 \div 4 =$



 Model and Solve.
 What do you notice?

 How many _____s are in ___?
 OR _____divided into _____equal parts . . . What is the value of each part?



CC Fraction Division Standards

5th Grade

CCSS.MATH.CONTENT.5.NF.B.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions using visual models.¹

¹ Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

6th Grade

Apply and extend previous understandings of multiplication and division to divide fractions by fractions using visual models and the relationship between multiplication and division.

CCSS.MATH.CONTENT.6.NS.A.1

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

For example, create a story context for (2/3) \div (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9 \dots$

7th Grade

CCSS.MATH.CONTENT.7.NS.A.2

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

CCSS.MATH.CONTENT.7.NS.A.3

Solve real-world and mathematical problems involving the four operations with rational numbers.¹

Common Core Standards Fraction Progression

Grade 4	Grade 5	Grade 6	Grade 7
Extend understanding of	Use equivalent fractions as a	Apply and extend previous	Apply and extend previous
fraction equivalence and	strategy for adding and subtracting	understandings of multiplication	understandings of operations
ordering using visual fraction	fractions (with unlike	and division to divide fractions by	with fractions to add, subtract,
models.	denominators, including mixed	fractions.	multiply, and divide rational
	numbers) using visual fraction		numbers.
Build fractions from unit	models and equations.	Interpret and compute quotients	
fractions by applying and		of fractions, and solve word	
extending previous	Apply and extend previous	problems involving division of	
understandings of operations	understandings of multiplication	fractions by fractions using visual	
on whole numbers using visual	and division to multiply and divide	fraction models, equations, and	
fraction models and equations.	fractions using visual fraction	the relationship between	
	models or equations. (multiply a	multiplication and division to	
Add and subtract fractions and	fraction or whole number by a	explain and represent problems.	
mixed numbers with like	fraction; divide unit fractions by		
denominators, using visual	whole numbers and whole		
fraction models and equations.	numbers by unit fractions)		
Apply previous understandings			
of multiplication to multiply a			
fraction by a whole number,			
using visual fraction models			
and equations.			

Number Talks

A HIGH LEVERAGE PRACTICE FOR DEVELOPING PROCEDURAL FLUENCY



Table Talk

What is a Number Talk?

What experiences have you had with Number Talks?

What are the benefits of a Number Talk?

Tell Me All You Can

$$\frac{2}{3}$$
 of 16

The answer will be greater than _____ because _____. The answer will be less than _____ because _____. The answer will be between _____ and _____ because _____. The answer will be about _____ because _____.

Another Type of Number Talk





The practice of number talks is one of the most powerful vehicles I know for helping students learn to reason with numbers and make mathematically convincing arguments, for building a solid foundation for algebraic reasoning, and for teaching mathematics as a sense-making process. If all teachers make this shift in their practice, it would represent a profound advancement in mathematics education.

Ruth Parker, Nationally Acclaimed Math Educator

Author: Making Number Talks Matter

Number Talk Co-Planning



Co-Planning and Preparing to Implement

1. Choose a problem for your students and find a partner who chose the same problem.

2. Plan your number talk using the "Planning a Number Talk" handout.

3. Practice leading the number talk with your partner.

Follow-Up

Implement in your class.

Take a picture of student strategies.

Jot some notes about your reflections.

Be ready to share in workshop 2

Upcoming Workshops

Operations & Algebraic Thinking/Expressions and Equations/Functions Nov. 9 (CVESD, SUHSD, SDUSD, Preuss, & Gompers) Dec. 3 (Vista)

Geometry

Jan. 7 (Vista)

Jan. 11 (CVESD, SUHSD, SDUSD, Preuss, & Gompers)