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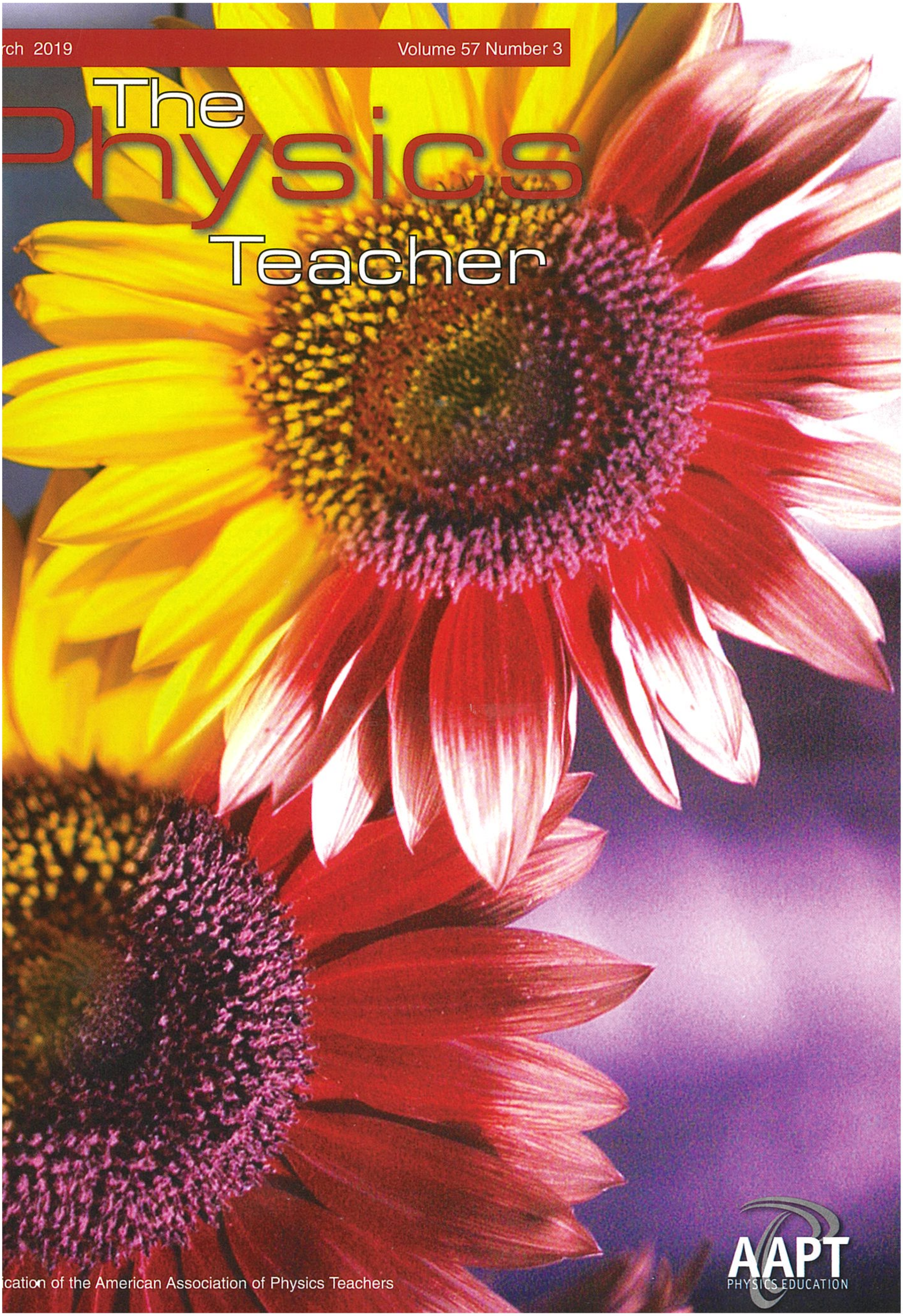

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AAPT
PHYSICS EDUCATION

Two Beakers, Five E's, Twenty Pennies, and Archimedes' Principle

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The lesson presented here provides a multifaceted inquiry-based opportunity to develop a deep grasp of Archimedes' principle. It was originally developed for high school teachers as part of their training in the basic structure of the Next Generation Science Standards. There are numerous demonstrations illustrating Archimedes' principle,[1-5] many of which can be found in this journal.[6-8] (Please see the online appendix for further references from this journal.)[9] However, we have a unique combination of ingredients, i.e., the particular puzzle it addresses and its associated historical context; the biological application; the accessible, hands-on collection and exploration of data; the powerful graphs encouraging physics-based explanations; and the ability to compare with an analytical solution. These materials brought together in one place provide teachers a novel and exceptionally rich recipe to bring to their classrooms.

In approaching this puzzle, we have chosen as scaffolding the BSCS 5E[10] framework. This inquiry-based approach embraces the idea that learners build new ideas on top of old ones through several phases of learning: Engage, Explore, Explain, Elaborate, and Evaluate. It is beyond the scope of this work to include summative assessments addressing conceptual flow and formative assessments that might include examples of science notebook entries, student work, or written quizzes. Nevertheless, we have included student questions throughout and frequent references to Fig. 1, which serves as our foundation.

The Engage phase begins with the simple question the puzzle poses: when a boat is floating in a pool and its anchor is tossed overboard, how does the water level in the pool change? The Engage phase also shares a historical account showing that even the most accomplished physicists are not always right in their initial thinking. At first, the students would be invited to suggest their own method to Explore. Through the teacher's guidance, they would collect and graph several data sets of displaced water as a function of the mass of the object causing the displacement. The Explain phase encourages the students to identify trends in the data and look for the underlying physics responsible for these trends. The Elaborate phase describes how dolphins adapt to their buoyancy challenges—air at water's surface but food below it. Finally, the Evaluate phase returns to the initial question proposed in the Engage phase, inviting students to make their prediction, explain their reasoning, test their prediction, and try again if necessary.

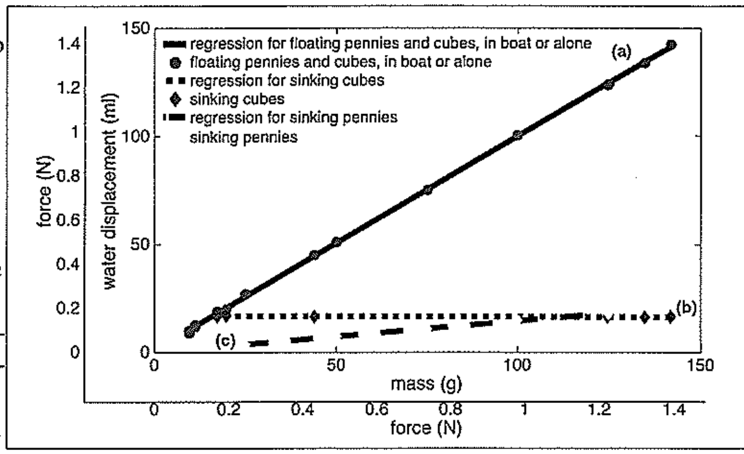

Fig. 1. Mass and corresponding water volume displacement for cubes and pennies placed directly in water and in a beaker floating in water. The solid regression line through the red data symbols (a) has a slope of 0.99 ml/g and an offset 0.96 ml. These symbols represent measurements of objects that floated either by themselves or within a beaker. The nearly flat dotted regression line (b), associated with the blue symbols representing measurements of cubes that sank when placed directly in water, has a slope of -0.0036 ml/g and an offset of 16.9 ml. The dashed regression line (c) through the gold symbols has a slope of 0.1453 ml/g and offset of 0.0817 ml. These symbols represent the measurements of the 10, 20, 30, 40, and 50 pennies that sank when put directly in water. A second pair of axis labels designates the equivalent force (N). The regression lines are all with respect to the inner axes. Note that equations for regression lines in terms of the second, outer axes will be different.

Engage

A celebrated buoyancy problem[3,7,12] among physicists goes something like the following: Consider an anchor in a boat floating in a pool. While the anchor is in the boat, the water level in the pool is marked. The anchor is then dropped to the bottom of the pool, with the boat still in the pool. The question is whether the water level in the pool goes up, down, or remains the same. As legend has it, this question was asked of George Gamow, Robert Oppenheimer, and Felix Bloch, all extraordinary physicists. To their embarrassment, they all got it wrong.[13] We provide here a hands-on desktop model of this puzzle that every student can systematically explore both by direct observation and through the amplifying lens of graphing data. For comparison's sake, our measurements as well as an analytical solution are included.

While others[1,7] have suggested different materials for variations of this classroom experiment, we suggest a simpler approach where the pool, boat, and anchor are represented, respectively, as an outer beaker, inner beaker, and 20 pennies. Water in the outer beaker corresponds to water in the pool.

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Beakers need to be chosen so that the inner one, with 20 pennies in it, will float within the outer one. The outer beaker should be small enough so that a change in water level can be sensibly estimated when the pennies are "tossed overboard." However, the space between the inner and outer beakers must be large enough that capillary effects do not confound water level readings. Similar reasoning was applied when the pennies are later replaced with cubes of different materials.

Explore

In which direction, by how much, and why the water level changes can be best explored by breaking the problem into two complementary investigations that are amenable to quantitative measurements. For the first query, objects are placed directly in a water-filled 50-ml beaker, and the change of water level is measured as a function of the mass of the object. Measurements are then repeated, recording the change in water level in a 500-ml outer beaker ("pool") for these same objects when they are placed within a 250-ml inner beaker ("boat") floating within the 500-ml beaker.

For each investigation two groupings of objects were chosen. The first group of objects were of the same density but different volumes, which was achieved by using 10, 20, 30, 40, and 50 pennies (all minted after 1982).[11] The second employed objects of the same volume but different densities, using 16.4-cm^3 cubes (Ajax Scientific) of solid aluminum, iron, copper, brass, nylon, pine, oak, and PVC. Note that the initial water level in the outer beaker is marked while the inner beaker is floating, empty, in the outer beaker.

Masses of the cubes were obtained using a digital stress-gauge scale that measured weight. After internally dividing by gravity, a digital stress-gauge scale (Scales Galore) that measured weight displayed readings of mass with a resolution of 0.1 g. Mass and volume of the pennies are 2.5 g and 0.35 cm^3, respectively.[11] Calibrated beakers allowed for estimates of the change in the water level. Marked in ml, changes in water level reflect a change in water volume equal to the volume of water displaced. Unless given, measurements of the mass and volume of the pennies and cubes and the associated displaced water volumes were repeated six times, averaged, and plotted. Details and recommendations for collecting measurements and equipment are provided in the online appendix,[9] where a table of measurements, and the development of an algebraic solution for the change in height, can also be found.

Graphing the mass (g) and water volume displacement (ml) measurements associated with the pennies and cubes when placed directly in the water and in the floating beaker provides a powerful avenue of exploration (Fig. 1). Plotted this way, we find that objects that float, both within a beaker and by themselves, essentially fall along a straight line [Fig. 1, regression line (a)] at 45° to the axes. Cubes of equal volume that sank formed a nearly horizontal line [Fig. 1, regression line (b)]. The corresponding values for the 10, 20, 30, 40, and 50 pennies at the bottom of the beaker appear on a line beneath that for floating objects and with a shallower slope [Fig. 1, regression line (c)].

Explain

"There is magic in graphs... the profile of a curve reveals in a flash whole situations... the curve informs the mind, awakens the imagination, convinces."
(Henry Hubbard 1939)

Students are encouraged to find data trends in Fig. 1 and identify the underlying physical laws that these trends reflect. In this regard, it will be helpful to recall that 1 ml of water by definition is equal to 1 cm^3 of water and has a mass essentially equal to 1 g.[14] Therefore, by decree, water has a density of 1, i.e., 1 g/1 cm^3 or 1 g/1 ml. Consequently, for any measurement of mass and displacement of water, it will fall along line (a).

Consider the solid regression line labeled (a) in Fig. 1 characterizing water displacement and mass measurements of both pennies and cubes that float either by themselves, e.g., pine and oak cubes, or in the 250-ml beaker (boat). This line has a slope of 0.99 ml/g and an offset 0.96 ml. Referring to the original (inner) axis labels, this line reflects, within the accuracy of our measurements, the simple observation that the volume of water, measured in ml, displaced by a floating object, whether by itself or in a container, is equal to the mass of the object, measured in g.

To better see the underlying physical law that this line reveals, substitute g for ml on the y-axis in Fig. 1 and multiply both axes by gravity. The units for both axes (outer axes) now become that of force, with the present data lying between 0.02 and 2.0 N (Fig. 1). In terms of these units the solid regression line (a) becomes $y = 0.009 + 0.99x$, with x and y in N. We find that the pennies and cubes will float when the buoyant force, which must equal the object's weight (x-axis), is equal to the weight of the water displaced (y-axis). This graphical relationship is a direct manifestation of Archimedes' principle, which on a more fundamental level is due to Newton's third law.[3,4] The weight of the floating object is sustained by the weight of the displaced water communicated through pressure.[3,4]

Furthermore, what might the 0.009-N offset imply? In addition to the buoyancy force, which changes with mass of the floating object, the offset represents a constant force. Such a force could arise as a result of surface tension between the inner beaker and the water, which in principle can be detected.[4]

The dotted regression line (b) through the points for cubes that sank has a near zero slope, −0.0036 ml/g and an offset of 16.9 ml, with respect to the inner axes. What does this line reflect? The dotted line shows that the amount of water displaced when the cube is totally submerged is equal to about 16.9 ml and is independent of the mass of the cube. Given that 1 ml is equal to 1 cm^3, this offset can be interpreted as 16.9 cm^3, which within our limits of uncertainty is close to 16.3 cm^3, the volume of the cube. For the essentially incompressible materials used here, volume is conserved.

Finally, consider the dashed linear regression line (c) in Fig. 1 that appears below the solid regression line (a). Line (c) is associated with the sunken pennies. With a slope of 0.145 ml/g, the difference between regression lines (a) and (c) increases with the mass of the pennies. This divergence means that the difference in water displacements between when the

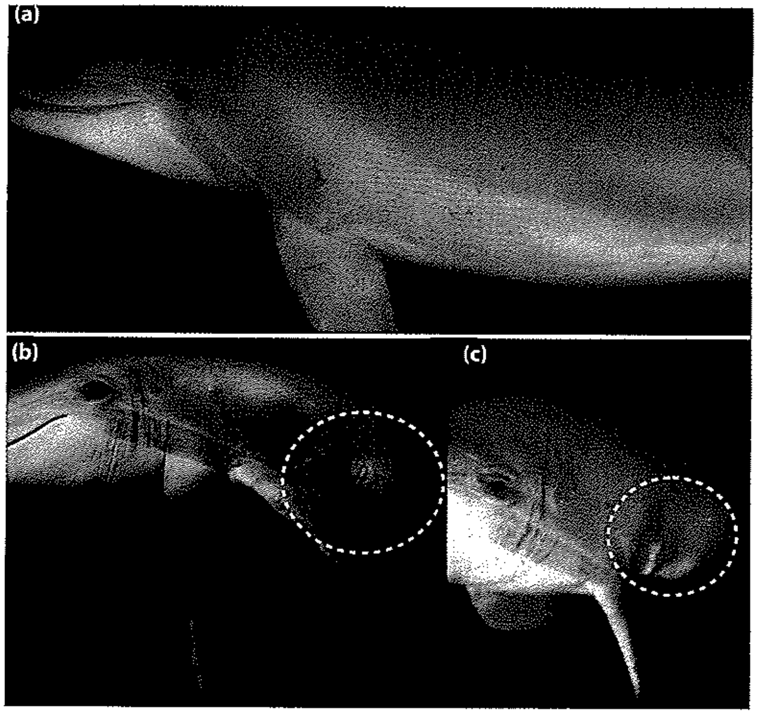

Fig. 2. In 1969, the Atlantic bottlenose dolphin Tuffy was trained to dive 300 m and press its rostrum to a paddle that activated a camera to take its picture. (a) Tuffy at surface, (b) and (c) Tuffy at 300 m depth; (b) and (c), the first dolphin selfies, revealed a dramatic collapse in its thoracic cavity (signified by dotted circles). (Photo courtesy of the U.S. Navy)

pennies are floating (a) in the 250-ml beaker (boat) or sunk (b) at the bottom of the 500-ml beaker (pool) will increase with additional pennies. If these data are replotted with the displaced volume as the independent variable, the new slope is 1/[0.145(ml/g)] or 6.90 g/ml, equivalent to 6.90 g/cm^3. As expected, within the accuracy of our measurements, this is nearly equal to the density of pennies, which is 7.2 g/cm^3.

Taking this information together, data points below regression line (a) correspond to objects that have a density greater than one. These objects will sink when placed directly in the water.

Elaborate

Dolphins have to contend with the persistent problem that their life-giving oxygen is only at the surface but their food is found at depth. Dolphins are essentially neutrally buoyant at the surface of the water. This buoyancy is achieved by filling their lungs with air to a volume of about 7000 ml,[15] nearly the volume of an NBA official basketball. Extrapolating from Fig. 1 [regression line (a)] or by invoking Archimedes' principle, we can surmise that the mass, which this volume of displaced water supports is about 7000 g. The corresponding buoyant force is approximately 70 N. Partly to mitigate this upward buoyant force while diving, the dolphin's flexible thorax (chest) begins to collapse. This collapse is why the image of the dolphin, Tuffy [Figs. 2(b) and (c)], appears so peculiar; there is a dramatic reduction of volume of the thorax behind the left flipper.

This adaptation of the rib cage serves two purposes. Foremost, it allows the lungs and the alveoli within them to collapse. The alveoli are tiny air sacs in the lungs where the exchange of oxygen and carbon dioxide takes place. Their collapse keeps pressurized nitrogen in the lungs from going into solution at depth, thereby avoiding nitrogen narcosis or bubbles in the blood and tissues during return to the surface.[15,16] Less critical but still important, this reduction in the dolphin's volume also results in a reduction in buoyancy. At a depth of 300 m [Figs. 2(b) and (c)], the collapse of the thoracic cavity is very pronounced, with a volume now of about 233 ml. From Fig. 1 line (a) or Archimedes' principle, this collapse results in a reduction in buoyancy to about 2.3 N.[14] This reduction of buoyancy allows the dolphin to sink so that it glides further when diving, increasing speed and saving energy.[17] Conversely, rising to the surface, the dolphin's thoracic cavity will expand, providing an additional upward, buoyant force.

Evaluate

With fresh insight, we now return to our original problem of the anchor tossed from a floating boat into a pool and ask if the pool's water level goes down, up, or stays the same. As the underlying physical principles are the same, it is exhilarating that we can investigate both the direction and corresponding volume change in the pool at our desks with only beakers and pennies. Moreover, this same logic can be applied to the launching of a ship and the rise of the ocean.[3]

Although the answer can be indirectly surmised from Fig. 1, let us now be more deliberate in our approach. With the empty 250-ml inner beaker (boat) floating in a water-filled 500-ml outer beaker (pool), we arbitrarily choose a reference water level of about 150 ml [Fig. 3(a)]. Placing 20 pennies (anchor) in the inner beaker, we observe that the water level on the outer beaker rises about 50 ml [Fig. 3(b)]. From Fig. 1, regression line (a), this water displacement corresponds to an upward buoyant force on the inner beaker of about 0.5 N, equal to the weight of the 20 pennies, i.e., 50 g/1000 kg * 9.8 m/s^2 = 0.49 N. Marking the water level on the outer beaker [Fig. 3(b)], we now have a desk-size setup of the original puzzle.

When the 20 pennies are transferred from the inner beaker and placed directly in the water of the 500-ml outer beaker [Fig. 3(c)], the new water level in the outer beaker is significantly lowered. However, relative to our original 150-ml reference water level, it is about 8 ml higher [Fig. 3(c)], consistent, within the accuracy allowed, with the 7.0-ml volume of the 20 pennies, i.e., 20 * 0.348 ml = 6.96 ml. Again this water displacement measurement is consistent with the previous data plotted in Fig. 1 [i.e., line (c)].

Since the total mass of the pennies is proportional to their number, we deduce from Fig. 1 that jettisoning any number of pennies from the inner beaker (boat) will result in the water level in the outer beaker's (pool) decreasing. Furthermore, the divergence between the (a) and (c) regression lines in Fig. 1 indicates that increasing the number of dumped pennies will result in a greater drop in the water level in the outer beaker. Curiously, although the water level in the outer beaker has decreased as a consequence of throwing the pennies overboard [Figs. 3(b) and (c)], the volume of water within the outer beaker has not changed. What accounts for the decrease in water level in the larger beaker? It is simply the compensating increase in the water depth beneath the smaller beaker after the pennies are tossed overboard.

Note, while collecting the sunken penny measurement data in Fig. 1, line (c), the inner beaker or boat was not included. To get sufficient resolution for the water displaced by the submerged pennies and cubes, we used a 50-ml pool, so including the 250-ml beaker (boat) was not an option. Nevertheless, this situation did not affect our final conclusion. As long as the boat is either always present or always absent, it does not matter since the displaced water associated with the boat's buoyancy does not change.

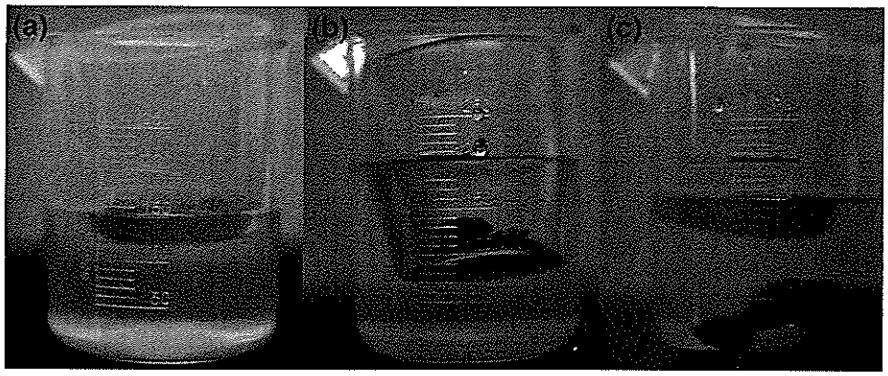

Fig. 3. Deconstructing the celebrated buoyancy problem: (a) establishing a 150-ml water level reference in the 500-ml beaker with an empty 250-ml beaker present; (b) water level in 500-ml beaker when 20 pennies are placed in the 250-ml beaker; water volume displacement is about 50 ml from (a); (c) water level when 20 pennies are submerged; water volume displacement is about 8 ml from (a). The difference in displaced water volumes between panels (b) and (c) is 42 ml, which amounts to a drop in water level on the outer beaker.

In summary, how the water level in the outer beaker changes when the 20 pennies are thrown overboard depends on whether the displaced water volume, relative to our 150-ml reference, is greater when the pennies are floating in the 250-ml beaker (50 ml) or when they are at the bottom of the 500-ml beaker (8 ml). Our answer is the difference in water levels, which corresponds to a net drop of about 42 ml and is consistent with the difference we find between regression lines (a) and (c) in Fig. 1 for a mass of 50 g, the mass of the pennies. The great power of the graphs in Fig. 1 is that for any number of pennies (albeit minted after 1982), one can calculate the change in water level in the 500-ml outside beaker (pool) when the pennies (anchor) are dumped out of the inner beaker (boat).

Students should be able to continue this line of reasoning for any number of pennies dumped in a 250-ml outer beaker using Fig. 1. For example, for 100 pennies, or 250 g, the extrapolated regression line for (a) yields 100.15 ml, and for (b) equals 14.61, the difference equaling 85.52 ml.

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