Tone association and output locality in non-linear structures

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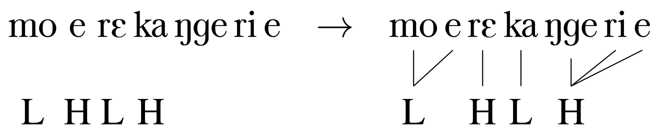
Overview

- Computational characterizations of phonology lead to restrictive, testable, and learnable theories of phonological processes (Heinz, 2018).
- **Research question**: what kind of maps are **tone mapping patterns**?



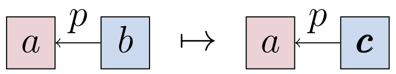
(Kikuyu; Clements and Ford 1979)

- **Result**: Maps defined with **quantifier-free least fixed point logic** give a restrictive, **output-local** characterization of tone mapping patterns

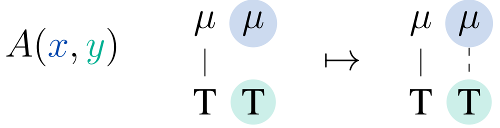
Logical maps

- Logical formulas define outputs through properties of the input (Courcelle, 1994)

$$c(x) \overset{\text{def}}{=} b(x) \wedge a(p(x))$$



- Tone mapping defines **association**

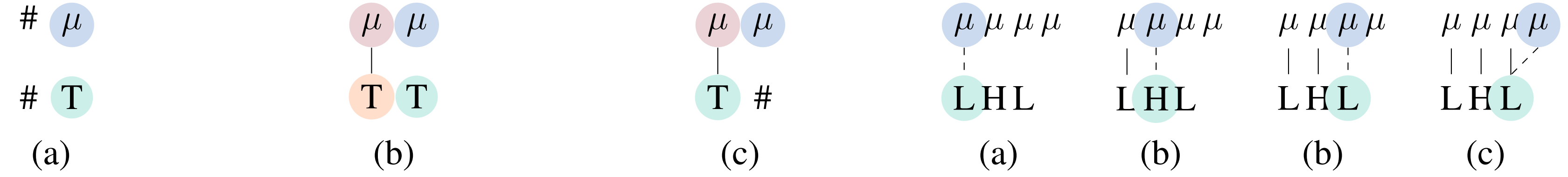


- These definitions
  - are **quantifier free** (Chandlee and Lindell, forthcoming)
  - use **least fixed point operators** (Libkin 2004), which allow recursive definitions (shown here with *implicit definitions*; Rogers 1997)
  - Use *either* predecessor (p) or successor (s)

Analyses

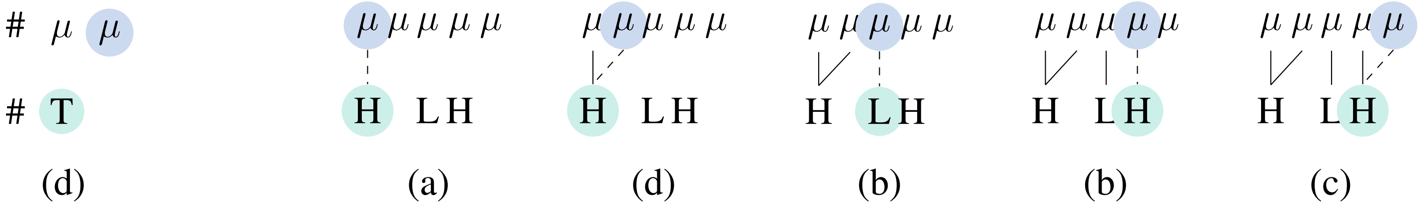
Mende (Left-to-right)

$$R(x,y) \overset{\text{d}}{=} \underbrace{\big(first(x) \wedge first(y)\big)} \vee \underbrace{\big(R(p(x),p(y))} \vee \underbrace{\big(last(y) \wedge R(p(x),y)\big)} \qquad A(x,y) \overset{\text{d}}{=} R(x,y)$$



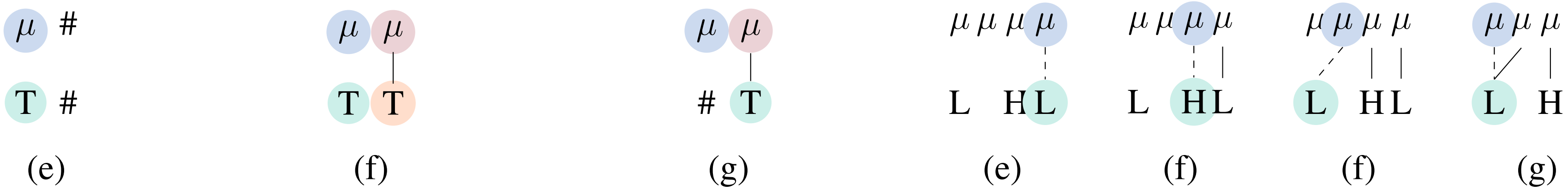
Kikuyu (1st tone to 1st and 2nd TBUs; then left-to-right)

$$R(x,y) \overset{\text{d}}{=} (\text{b}) \vee (\text{c}) \vee \underbrace{\big(first(y) \wedge second(x)\big)} \qquad A(x,y) \overset{\text{d}}{=} (\text{a}) \vee R(x,y)$$



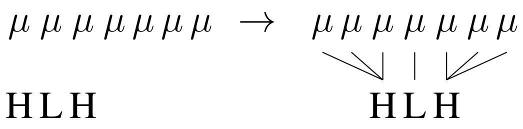
Hausa (Right-to-left)

$$R(x,y) \overset{\text{d}}{=} \underbrace{\big(last(x) \wedge last(y)\big)} \vee \underbrace{\big(R(s(x),s(y))\big)} \vee \underbrace{\big(first(y) \wedge R(s(x),y)\big)} \qquad A(x,y) \overset{\text{d}}{=} R(x,y)$$



Discussion

- Begins solution for problem of logical complexity of tone mapping Jardine (2017)
- A principled characterization of the range of possible tone association patterns
- Can capture patterns that cannot be captured by OT ALIGN constraints
- Explains absence of unattested patterns, like centering:



- Recursive definitions provide the first logical definition of output-based locality for phonology

Select References

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